2893 [1921, 184]. **Proposed by NATHAN ALTSHILLER-COURT, University of Oklahoma.**

Find the locus of the mid-point of the segment determined by two given skew lines in a variable plane turning about a fixed axis, not coplanar with either of the given lines.

SOLUTION BY J. K. WHITTEMORE, Yale University.

Let the axis of the variable plane be the z axis, and suppose the x and y axes chosen parallel one to each of the given skew lines. This coördinate system is generally oblique. The equations of the skew lines are (1) $y = b_1$, $z = c_1$; (2) $x = a_2$, $z = c_2$; the variable plane, $y = \lambda x$, intersects the two lines in points of coördinates, (1) $x = b_1/\lambda$, $y = b_1$, $z = c_1$; (2) $x = a_2$, $y = \lambda a_2$, $z = c_2$. The coördinates of the mid-point of the segment, a point of the required locus, are

$$x = \frac{a_2}{2} + \frac{b_1}{2\lambda}, \qquad y = \frac{b_1}{2} + \frac{\lambda a_2}{2}, \qquad z = \frac{c_1 + c_2}{2}.$$

Eliminating the parameter λ, the equations of the required locus are

$$\left(x - \frac{a_2}{2}\right)\left(y - \frac{b_1}{2}\right) = \frac{a_2 b_1}{4}, \qquad z = \frac{c_1 + c_2}{2}.$$

The locus is a hyperbola lying in a plane parallel to both given skew lines and half way between them; its asymptotes are parallel to the given skew lines; its center is the point of intersection of the diagonals of the parallelopiped, three of whose edges lie on the axis of the variable plane and the given skew lines; it intersects the axis of the variable plane.

Also solved by WILLIAM HOOVER.

2900 [1921, 277]. **Proposed by I. A. BARNETT, University of Saskatchewan.**

AB is the diameter of a circle and Q_0 any point on the circumference; $Q_1, Q_2, Q_3, \cdots$, are the points of bisection of the arcs $AQ_0, AQ_1, AQ_2, \cdots$; to prove that the product of the chords of the circle $BQ_1, BQ_2, BQ_3, \cdots, BQ_n$ is equal to $OA^n \cdot (AQ_0/AQ_n)$, O being the center of the circle.

SOLUTION BY A. M. HARDING, University of Arkansas.

Arc AQ_1 = arc Q_1Q_0. Hence, $\angle OBQ_1 = \angle AQ_0Q_1 = \angle Q_1AQ_0$. Hence, the isosceles triangles, OBQ_1 and Q_1Q_0A are similar. Then,

$$\frac{BQ_1}{AQ_0} = \frac{OB}{AQ_1}; \qquad \text{that is,} \qquad BQ_1 = OA \cdot \frac{AQ_0}{AQ_1}.$$

In a similar manner, it may be shown that

$$BQ_2 = OA \cdot \frac{AQ_1}{AQ_2},$$
$$BQ_3 = OA \cdot \frac{AQ_2}{AQ_3},$$
$$\vdots$$
$$BQ_n = OA \cdot \frac{AQ_{n-1}}{AQ_n}.$$

Multiplying these equations gives

$$BQ_1 \cdot BQ_2 \cdot BQ_3 \cdot \cdots BQ_n = OA^n \cdot \frac{AQ_0}{AQ_n}.$$

Also solved by T. M. BLAKSLEE, ARTHUR PELLETIER, and A. V. RICHARDSON.

2945 [1922, 29]. **Proposed by T. M. BLAKSLEE, Ames, Iowa.**

A point P in the plane of the triangle ABC rotates in a given direction around the vertices taken in either cyclical order, in each case through an angle equal to the corresponding angle of the triangle. That is, for example, AP rotates around A through an angle equal to the angle A of the triangle; then BP around B through an angle equal to the angle B, and so on. Prove